Math in Basketball: Try other basketball challenges

ANSWER KEY

In this challenge, you get to choose a new set of player stats, then use the 3 key variables to figure out the maximum height the ball reaches during a free throw shot.

(This activity can also be completed online. Go to www.getthemath.org, click on "The Challenges," then scroll down and click on "Math in Basketball: Try other challenges.")

FAST BREAK FACTS: KNOW THE STATS

1. **Identify what you already know.** Look at the **Fast Break Facts** on the last page of this handout for information about the 3 key variables and select a player's stats from the choices below:

   - The Acceleration of Gravity: ________
   - Initial Vertical Velocity (Select one): ___ 20 ft/sec ___22 ft/sec __ 24 ft/sec
   - Release Height (Select one): ____ 5 ft ____ 6 ft ____ 8 ft

   Combine these 3 key variables used to calculate the ball's height, *h*, at a given time, *t*, by setting up an equation to get started.

   h(*t*) = ______________________________

   The answer will vary depending on the stats chosen by the student, but will follow this model:
   $h(t) = -16t^2 + v_0t + h_0$ (Note: Initial Vertical Velocity = v_0; Release Height = h_0)

   For example, if the student selected Initial Vertical Velocity = 5 ft and Release Height = 20 ft/sec, the correct equation would be $h(t) = -16t^2 + 20t + 5$.

AT WHAT TIME(S) DOES THE BALL REACH 10 FEET?

2. **Plan it out.** What strategy will you use? Select one or more representations, such as your equation or a graph (found on the last page), to **calculate the value(s) of *t* when the ball reaches a height of 10 feet.**

Strategy A:

The height (h) of the ball, in feet, at a given time (t) is represented by the equation:

$h(t) = -16t^2 + v_0t + h_0$

[Replace initial vertical velocity and release height values based on selection above.]

The value for t at 10 feet would occur at two points in time, one on the way up, the other at the hoop.

Substitute 10 feet for h(t) and solve.

Write in standard form: $0 = at^2 + bt + c$ by subtracting 10 from each side.

Solve algebraically using the quadratic formula, $t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, or complete the square to find two values of t.

Strategy B:

Another option is to graph the equation for the height of the ball, either using a graphing calculator or paper and pencil with a table of values. Then, you can use your graph to estimate the values of t at which the ball reaches 10 feet.

3. **Solve your problem**. Show all your steps. You may use the graph on the last page of this handout or show your work in the space below.
   See below for all solutions.

AT WHAT TIME(S) DOES THE BALL REACH THE MAXIMUM HEIGHT?

4. **Plan it out.** What strategy will you use? Select one or more representations, such as your equation or a graph (found on the last page), to **calculate the value(s) of *t* when the ball reaches its maximum height.**

Strategy A:

Represented graphically, the equation for height as a function of time, or h(t), is a parabola. Like all parabolas, it is symmetrical, meaning that it has an axis of symmetry that passes through the vertex, or highest point. Since you now know the two values of t when the ball reaches a height of 10 feet, you can find the axis of symmetry by calculating the halfway point, or mean, between these two times. This will give you the value of t when the ball reaches its maximum height.

Strategy B:

You can use the properties of the graph of the equation for h(t) to find the value of t for the vertex or maximum point. For a parabolic function of the form $0 = at^2 + bt + c$, where $a \neq 0$, the value for time (t) is represented by the x-coordinate of the vertex $\left(-\frac{b}{2a}\right)$ of the parabola.

5. **Solve your problem**. Show all your steps. You may use the graph on the last page of this handout or show your work in the space below.
   See below for all solutions.

WHAT IS THE MAXIMUM HEIGHT OF THE BASKETBALL?

6. **Plan it out.** What strategy will you use? Select one or more representations, such as your equation or a graph (found on the last page), to **calculate the maximum height the ball will reach on its way to the basket.**

   Using the value of *t*, or time, when the ball reaches its maximum height, you can substitute that value into the equation you set up for h(*t*) to find the height of the ball at that time, or use graphical representation.

7. **Solve your problem.** Show all your steps. You may use the graph on the last page of this handout or show your work in the space below.
   See below for all solutions.

ALL FINAL SOLUTIONS:
[h_0 = Release Height and v_0 = Initial Vertical Velocity]

h_0 = 5' and v_0 = 20 ft/sec

$h(t) = -16t^2 + 20t + 5$

At what times (in sec) does the ball reach 10 feet? $t = 0.35$ $t = 0.90$

At what time does the ball reach its maximum height? $t = 0.63$ (or 5/8 sec)

Maximum height = 11.25 feet

h_0 = 5' and v_0 = 22 ft/sec

$h(t) = -16t^2 + 22t + 5$

At what times (in sec) does the ball reach 10 feet? $t = 0.29$ $t = 1.09$

At what time does the ball reach its maximum height? $t = 0.69$ (or 11/16 sec)

Maximum height = 12.56 feet

h_0 = 5' and v_0 = 24 ft/sec

$h(t) = -16t^2 + 24t + 5$

At what times (in sec) does the ball reach 10 feet? $t = 0.25$ $t = 1.25$

At what time does the ball reach its maximum height? $t = 0.75$ (or ¾ sec)

Maximum height = 14 feet

---

h_0 = 6' and v_0 = 20 ft/sec

$h(t) = -16t^2 + 20t + 6$

At what times (in sec) does the ball reach 10 feet? $t = 0.25$ $t = 1.0$

At what time does the ball reach its maximum height? $t = 0.63$ (or 5/8 sec)

Maximum height = 12.25 feet

h_0 = 6' and v_0 = 22 ft/sec

$h(t) = -16t^2 + 22t + 6$

At what times (in sec) does the ball reach 10 feet? $t = 0.22$ $t = 1.16$

At what time does the ball reach its maximum height? $t = 0.69$ (or 11/16 sec)

Maximum height = 13.56 feet

Math in Basketball: Try other challenges *Answer Key*

h_0 = 6' and v_0 = 24 ft/sec

$h(t) = -16t^2 + 24t + 6$

At what times (in sec) does the ball reach 10 feet? $t = 0.19$ $t = 1.31$

At what time does the ball reach its maximum height? $t = 0.75$ (or ¾ sec)

Maximum height = 15 feet

---

h_0 = 8' and v_0 = 20 ft/sec

$h(t) = -16t^2 + 20t + 8$

At what times (in sec) does the ball reach 10 feet? $t = 0.11$ $t = 1.14$

At what time does the ball reach its maximum height? $t = 0.63$ (or 5/8 sec)

Maximum height = 14.25 feet

h_0 = 8' and v_0 = 22 ft/sec

$h(t) = -16t^2 + 22t + 8$

At what times (in sec) does the ball reach 10 feet? $t = 0.10$ $t = 1.28$

At what time does the ball reach its maximum height? $t = 0.69$ (or 11/16 sec)

Maximum height = 15.56 feet

h_0 = 8' and v_0 = 24 ft/sec

$h(t) = -16t^2 + 24t + 8$

At what times (in sec) does the ball reach 10 feet? $t = 0.09$ $t = 1.41$

At what time does the ball reach its maximum height? $t = 0.75$ (or ¾ sec)

Maximum height = 17 feet

FAST BREAK FACTS

THE 3 KEY VARIABLES

- **The Acceleration of Gravity –** which causes a ball to speed up, or accelerate, when falling at a rate of -32ft/sec^2. Use only downward pull or half of -32ft/sec^2, which is $-16t^2$.
- **Initial Upward Velocity (v_0)** - the angle and speed when it leaves the player's hand. Multiply by time to get the vertical distance traveled.
- **Release Height (h_0)** - the starting position of the ball.

PLAYER'S STATS (Select one of each)

Initial Upward Velocity: ___ 20 ft/sec ___ 22 ft/sec ___ 24 ft/sec

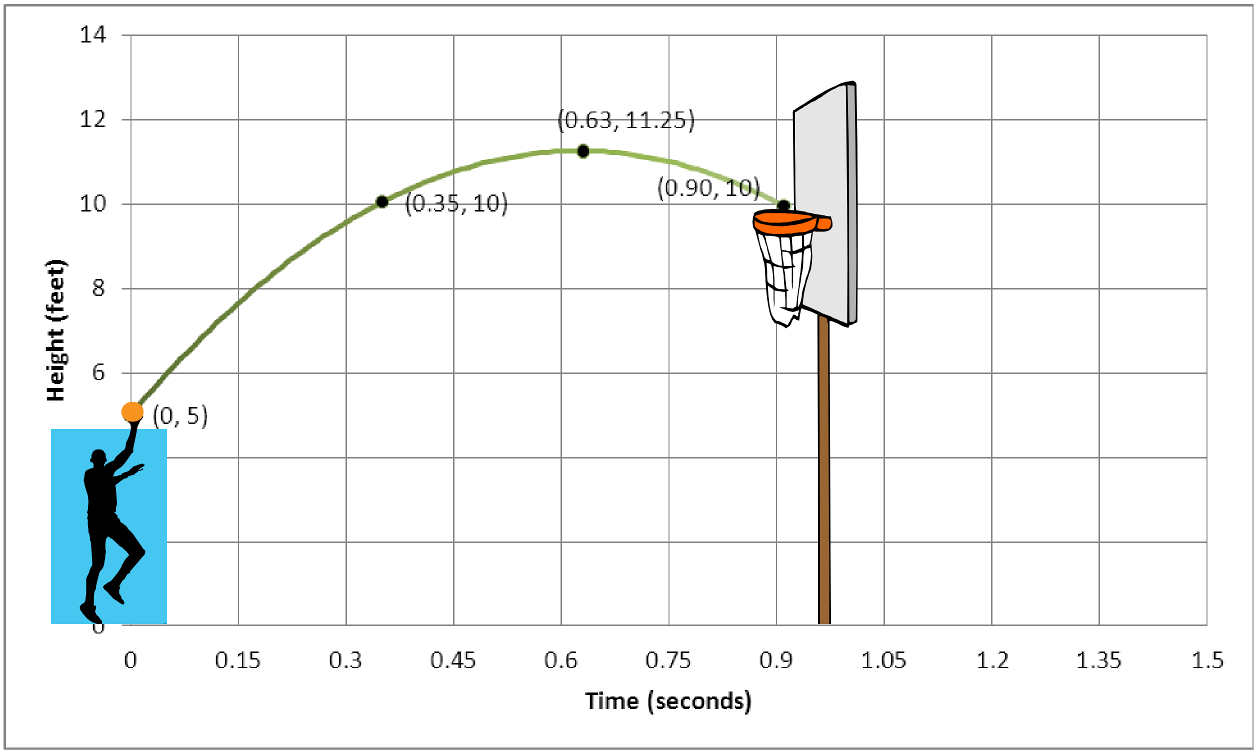
Release Height: ___ 5 ft ___ 6 ft ___ 8 ft

STANDARD COURT MEASUREMENTS

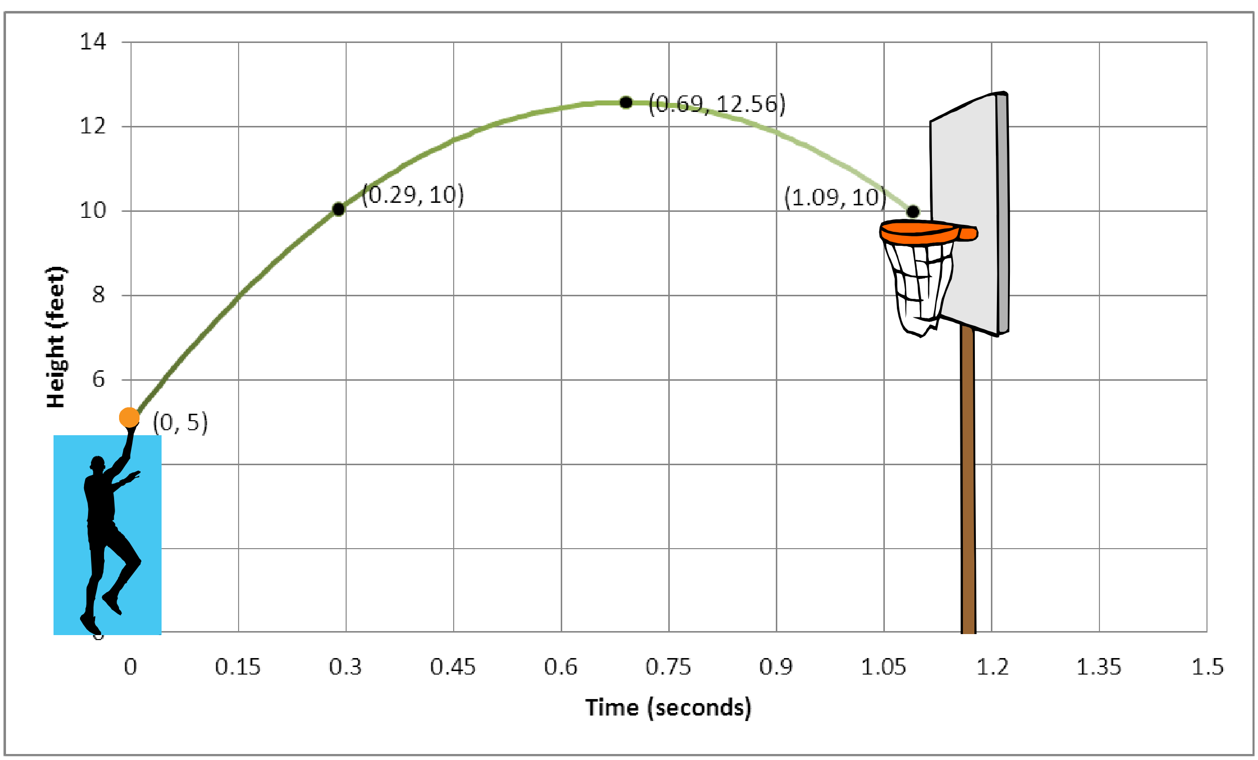
Height of the basketball hoop off the floor:	10 ft
Distance from the free throw line to backboard	15 ft
Diameter of hoop/rim	18 in

GRAPH YOUR DATA

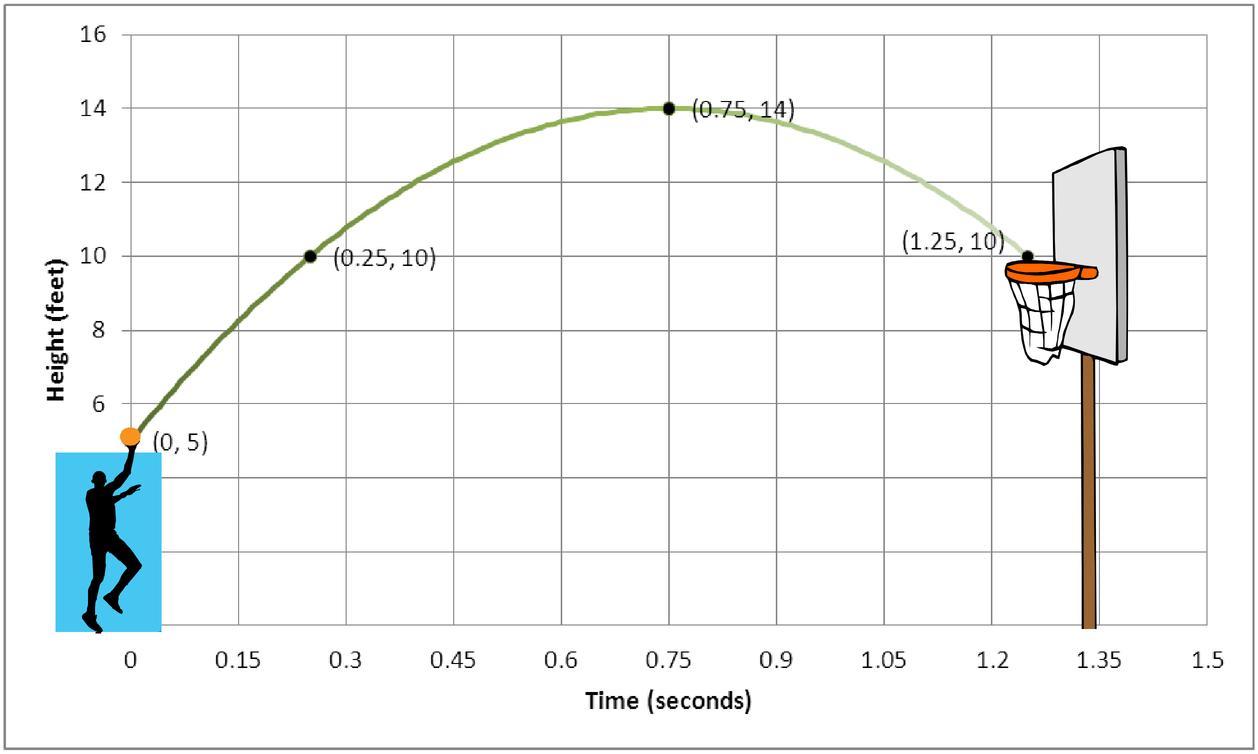
Release height = 5 ft, Initial Vertical Velocity = 20 ft/sec



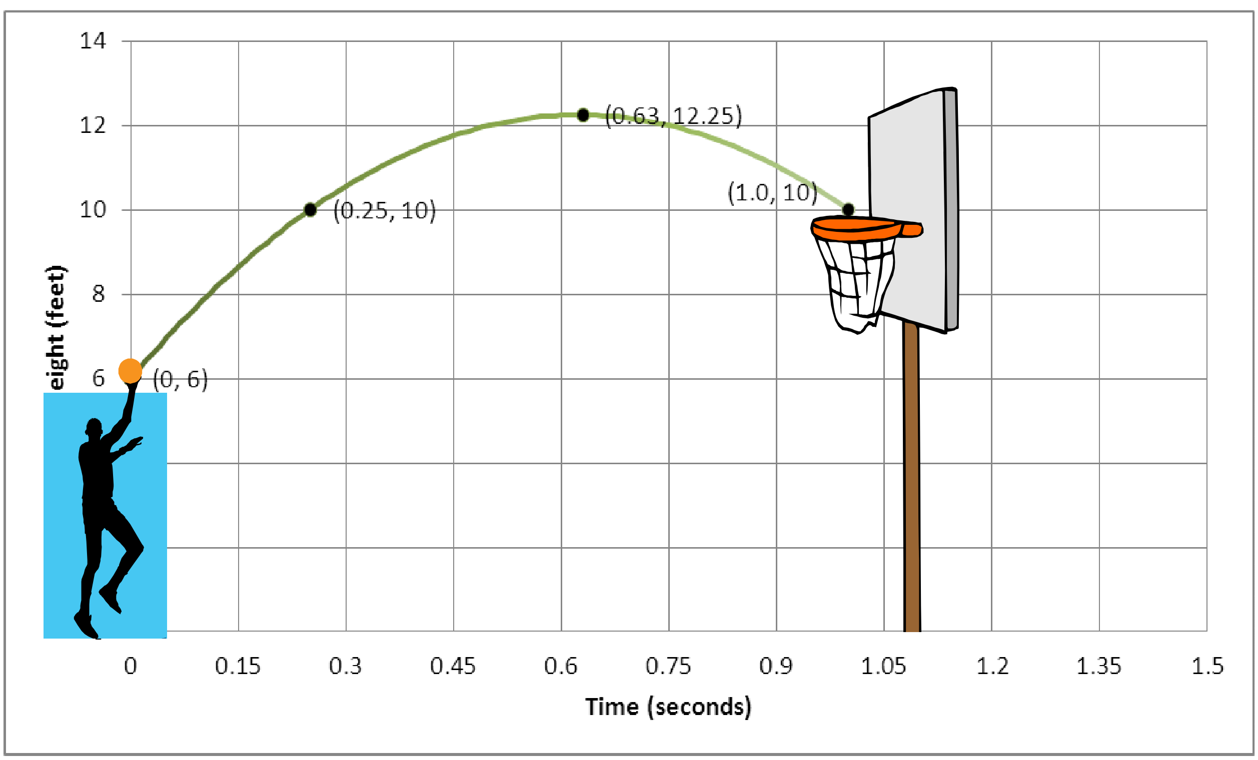
Release height = 5 ft, Initial Vertical Velocity = 22 ft/sec



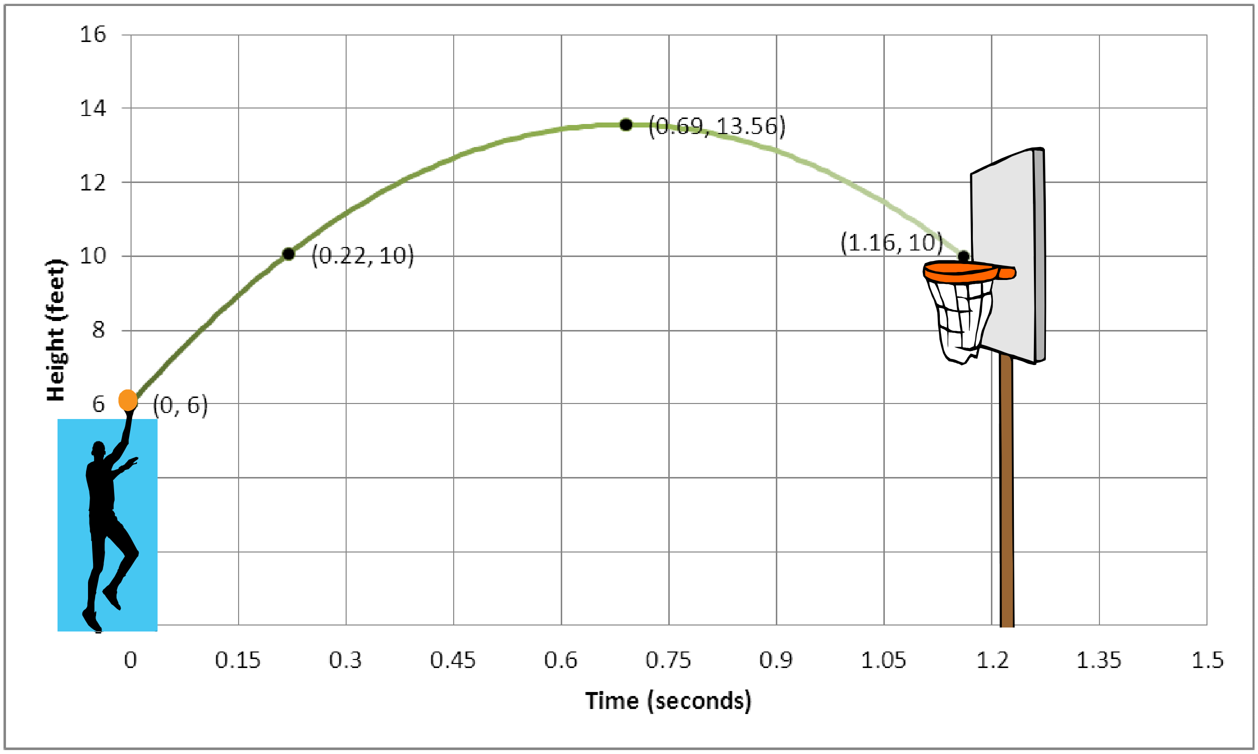
Release height = 5 ft, Initial Vertical Velocity = 24 ft/sec



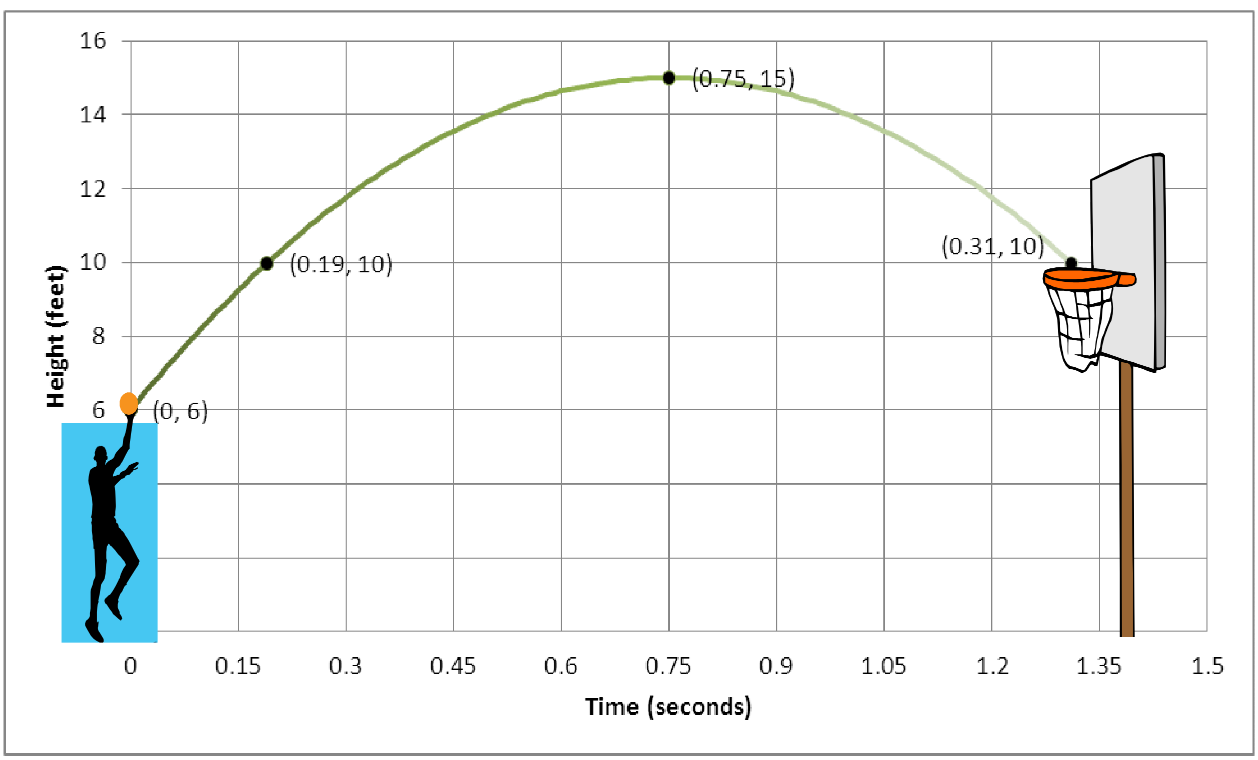
Release height = 6 ft, Initial Vertical Velocity = 20 ft/sec



Release height = 6 ft, Initial Vertical Velocity = 22 ft/sec

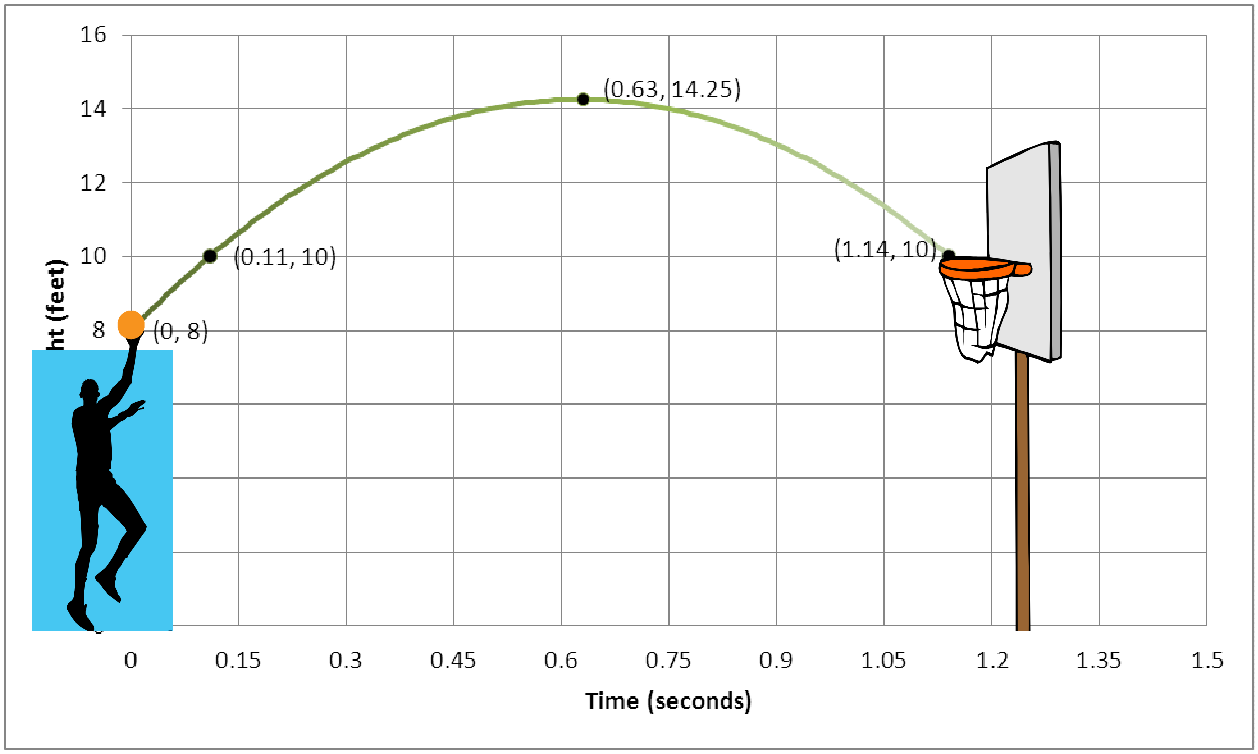


Release height = 6 ft, Initial Vertical Velocity = 24 ft/sec

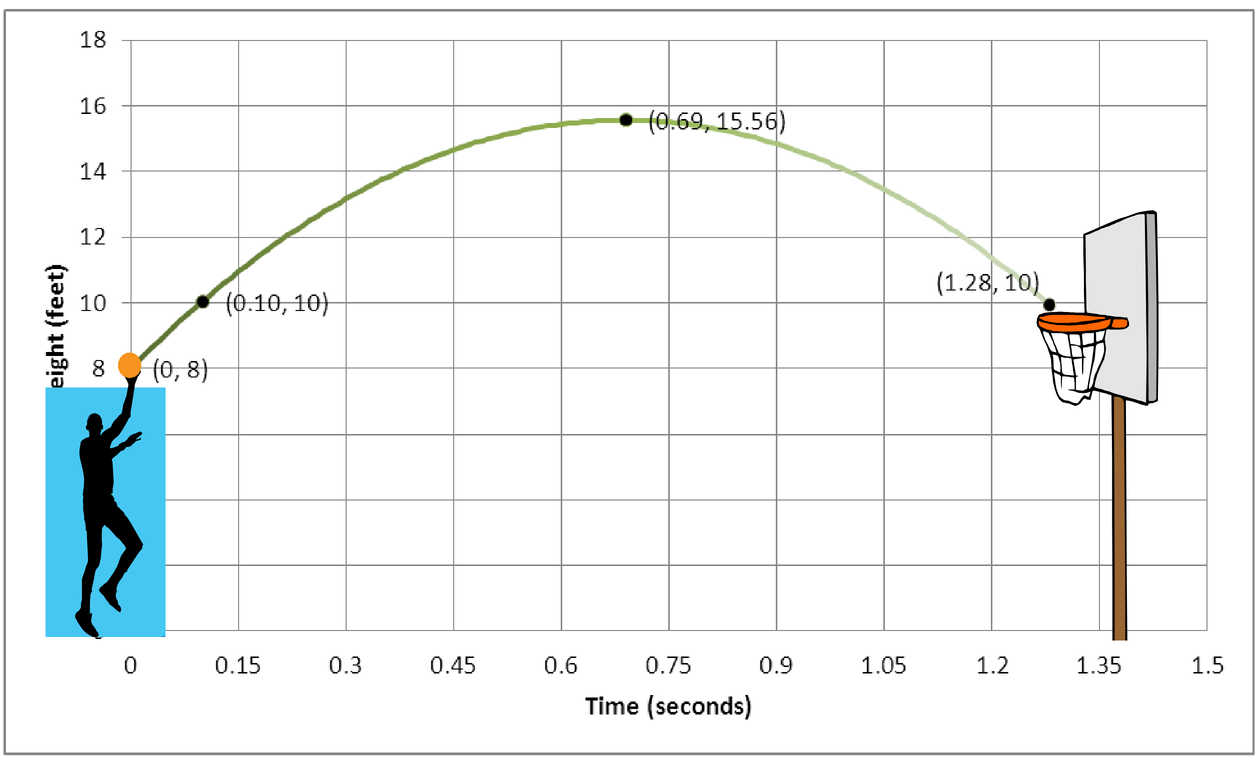


Math in Basketball: Try other challenges *Answer Key*

Release height = 8 ft, Initial Vertical Velocity = 20 ft/sec



Release height = 8 ft, Initial Vertical Velocity = 22 ft/sec



Release height = 8 ft, Initial Vertical Velocity = 24 ft/sec