Math in Basketball: Take the Challenge

ANSWER KEY

When NBA player Elton Brand steps to the free throw line, a number of key variables can influence his shot. Your challenge is to use the 3 key variables and Elton's stats to figure out the maximum height the ball reaches on its way into the basket to make the shot.

(This activity can also be completed online. Go to www.getthemath.org, click on "The Challenges," then scroll down and click on "Math in Basketball: Take the Challenge.")

FAST BREAK FACTS: KNOW THE STATS

1. **Identify what you already know.** Look at the Fast Break Facts (following the last question in this handout) for information about the 3 key variables and Elton's stats.

- The Acceleration of Gravity: **$-16t^2$**
- Elton's Initial Vertical Velocity : **24 ft/sec**
- Elton's Release Height: **7 feet**

Combine these 3 key variables used to calculate the ball's height, *h*, at a given time, *t*, by setting up an equation to get started.

$h(t) = -16t^2 + 24t + 7$

AT WHAT TIME(S) DOES THE BALL REACH 10 FEET?

2. **Plan it out.** What strategy will you use? Select one or more representations, such as your equation or a graph (found on the last page), to **calculate the value(s) of *t* when the ball reaches a height of 10 feet.**

Strategy A:

The height (h) of the ball, in feet, at a given time (t) is represented by the equation:

$h(t) = -16t^2 + 24t + 7$

where 24 is the initial vertical velocity and 7 is the release height

The value for t at 10 feet would occur at two points in time, one on the way up, the other at the hoop.

Substitute 10 feet for h(*t*): $10 = -16t^2 + 24t + 7$

Write in standard form: $0 = at^2 + bt + c$ by subtracting 10 from each side:

$0 = -16t^2 + 24t + (-3)$; *where* a = -16, b = 24, and c = -3

Solve algebraically using the quadratic formula, $t = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$, or complete the square to find two values of t.

Strategy B:
Another option is to graph the equation for the height of the ball, either using a graphing calculator or paper and pencil with a table of values. Then, you can use your graph to estimate the values of *t* at which the ball reaches 10 feet.

3. **Solve your problem**. Show all your steps. You may use the graph on the last page or show your work in the space below.

Strategy A:

- Use the quadratic formula: $t = \frac{-24 \pm \sqrt{24^2 - 4(-16)(-3)}}{2(-16)} = \frac{3 \pm \sqrt{6}}{4}$

$$\approx \mathbf{0.14 \text{ or } 1.36}$$

- Complete the square:

$$\begin{aligned} \mathbf{h}(t) &= -16t^2 + 24t + 7 \\ 10 &= -16t^2 + 24t + 7 \\ 3 &= -16t^2 + 24t \\ 3 &= -16(t^2 + \tfrac{3}{2}t) \\ \frac{3}{-16} &= (t^2 + \tfrac{3}{2}t) \\ \frac{3}{-16} + (-\tfrac{3}{4})^2 &= t^2 + \tfrac{3}{2}t + (\tfrac{1}{2} * (-\tfrac{3}{2}))^2 \\ \frac{6}{16} &= (t - \tfrac{3}{4})^2 \\ t &= \tfrac{3}{4} \pm \tfrac{1}{4}\sqrt{6} \approx 1.36 \text{ or } 0.14 \end{aligned}$$

Strategy B: See last page of this answer key for sample graph.

Your solution: (Round your answer to the nearest hundredth.)

- The time(s) the ball will reach 10 feet are: **0.14 and 1.36 seconds**

AT WHAT TIME DOES THE BALL REACH THE MAXIMUM HEIGHT?

4. **Plan it out**. What strategy will you use? Select one or more representations, such as your equation or a graph (found on the last page), to **calculate the value(s) of *t* when the ball reaches its maximum height**.

Strategy A:
Represented graphically, the equation for height as a function of time, or h(*t*), is a parabola. Like all parabolas, it is symmetrical, meaning that it has an axis of symmetry that passes through the vertex, or highest point. Since you now know the two values of *t* when the ball reaches a height of 10 feet, you can find the axis of symmetry by calculating the halfway point, or mean, between these two times. This will give you the value of *t* when the ball reaches its maximum height.

Strategy B:
You can use the properties of the graph of the equation for h(*t*) to find the value of *t* for the vertex or maximum point. For a parabolic function of the form $\mathbf{0 = at^2 + bt + c}$, where $a \neq 0$, the value for time *(t)* is represented by the x-coordinate of the vertex $\left(-\frac{b}{2a}\right)$ of the parabola.

5. **Solve your problem**. Show all your steps. You may use the graph on the last page or show your work in the space below.

Strategy A: Finding the mean between the two times (0.14 + 1.36) ÷ 2 = **0.75**
Strategy B: Using $\left(-\frac{b}{2a}\right) = \left(-\frac{24}{2*(-16)}\right) = \left(\frac{24}{32}\right) = \frac{3}{4}$

Your solution: (Round your answer to the nearest hundredth.)

- The time the ball will reach the maximum height is: **$\frac{3}{4}$ or .75 seconds**

WHAT IS THE MAXIMUM HEIGHT OF THE BASKETBALL?

6. **Plan it out**. What strategy will you use? Select one or more representations, such as your equation or a graph (found on the last page), to **calculate the maximum height the ball will reach on its way to the basket**.

Using the value of *t*, or time, when the ball reaches its maximum height, you can substitute that value into the equation you set up for h*(t)* to find the height of the ball at that time, or use graphical representation.

7. **Solve your problem**. Show all your steps. You may use the graph on the last page or show your work in the space below.

t = 0.75 seconds
$h(t) = -16(0.75)^2 + 24(0.75) + 7$
$h(t)$ = 16 feet

Your solution: (Round your answer to the nearest whole number.)

- The maximum height of the basketball will be at: **16 feet**

8. **What strategy would you use if you had to determine the maximum height for another player's release height and initial vertical velocity stats?** If you were going to email Elton Brand to **explain your process**, what would you tell him?

FAST BREAK FACTS

THE 3 KEY VARIABLES

- **The Acceleration of Gravity –** which causes a ball to speed up, or accelerate, when falling at a rate of -32ft/sec^2. Use only downward pull or half of -32ft/sec^2, which is -16t^2.
- **Initial Upward Velocity (v_0)** - the angle and speed when it leaves the player's hand. Multiply by time to get the vertical distance traveled.
- **Release Height (h_0)** - the starting position of the ball.

ELTON BRAND STATS

Elton's Average Initial Upward Velocity:	24 ft/sec
Elton's Average Release Height:	7.0 ft

STANDARD COURT MEASUREMENTS

Height of the basketball hoop off the floor:	10 ft
Distance from the free throw line to backboard	15 ft
Diameter of hoop/rim	18 in

GRAPH YOUR DATA