LESSON TITLE: Math in Special Effects (by Deborah L. Ives, Ed.D)

GRADE LEVEL/COURSE: Grades 7-10 Algebra

TIME ALLOTMENT: Two 45-minute class periods

OVERVIEW

Using video segments and web interactives from ***Get the Math,*** students engage in an exploration of mathematics, specifically reasoning and sense making, to solve real world problems and learn how special effects designers use math in their work. In this lesson, students focus on understanding the Big Ideas of Algebra: patterns, relationships, equivalence, and linearity; learn to use a variety of representations, including modeling with variables; build connections between numeric and algebraic expressions; and use what they have learned previously about number and operations, measurement, proportionality, and discrete mathematics as applications of algebra. Methodology includes guided instruction, student-partner investigations, and communication of problem-solving strategies and solutions.

In the Introductory Activity, students view a video segment in which they learn how Jeremy Chernick, a designer at J & M Special Effects, uses math in his work as he presents a mathematical challenge connected to a high-speed effect from a music video. In Learning Activity 1, students solve the challenge that Jeremy posed in the video, which involves using algebraic concepts and reasoning to figure out the relationship between light intensity and distance from a light source in order to help fix an underexposed shot. As students solve the problem, they have an opportunity to use an online simulation to find a solution. Students summarize how they solved the problem, followed by a viewing of the strategies and solutions used by the ***Get the Math*** teams. In Learning Activity 2, students try to solve additional interactive lighting (inverse relationship) challenges. In the Culminating Activity, students reflect upon and discuss their strategies and talk about the ways in which algebra can be applied in the world of special effects, lighting, and beyond.

LEARNING OBJECTIVES

Students will be able to:

- Describe scenarios that require special effects technicians to use mathematics and algebraic reasoning in lighting and high-speed photography.
- Identify a strategy and create a model for problem solving.
- Recognize, describe, and represent inverse relationships using words, tables, numerical patterns, graphs, and/or equations.
- Learn to recognize and interpret inverse relationships and exponential functions that arise in applications in terms of a context, such as light intensity.
- Compare direct and inverse variation.
- Solve real-life and mathematical problems involving the area of a circle.

MEDIA RESOURCES FROM THE *GET THE MATH* WEBSITE

www.getthemath.org

- The Setup **(video)** ***Optional***
  An introduction to ***Get the Math*** and the professionals and student teams featured in the program.
- Math in Special Effects: Introduction **(video)**
  Jeremy Chernick, special effects designer, describes how he got involved in special effects, gives an introduction to the mathematics used in high speed photography, and poses a related math challenge.
- Math in Special Effects: Take the challenge **(web interactive)**
  In this interactive activity, users try to solve the challenge posed by Jeremy Chernick in the introductory video segment.
- Math in Special Effects: See how the teams solved the challenge **(video)**
  The teams use algebra to solve the Special Effects challenge in two distinct ways.
- Math in Special Effects: Try other challenges **(web interactive)**
  This interactive provides users additional opportunities to use key variables and different lens sizes to solve related problems.

MATERIALS/RESOURCES

For the class:

- Computer, projection screen, and speakers (for class viewing of online/downloaded video segments)
- One copy of the "Math in Special Effects: Take the challenge" answer key
- One copy of the "Math in Special Effects: Try other challenges" answer key

For each student:

- One copy of "Math in Special Effects: Take the challenge" handout.
- One copy of the "Math in Special Effects: Try other challenges" handout
- One graphing calculator (optional)
- Rulers, grid paper, chart paper, whiteboards/markers, overhead transparency grids, or other materials for students to display their math strategies used to solve the challenges in the Learning Activities
- Colored sticker dots and markers of two different colors (optional)
- Computers with internet access for Learning Activities 1 and 2 (optional)
  (Note: These activities can either be conducted with handouts provided in the lesson and/or by using the web interactives on the ***Get the Math*** website.)

BEFORE THE LESSON

Prior to teaching this lesson, you will need to:

- Preview all of the video segments and web interactives used in this lesson.
- Download the video clips used in the lesson to your classroom computer(s) or prepare to watch them using your classroom's internet connection.

- Bookmark all websites you plan to use in the lesson on each computer in your classroom. Using a social bookmarking tool (such as delicious, diigo, or portaportal) will allow you to organize all the links in a central location.
- Make one copy of the "Math in Special Effects: Take the challenge" and "Math in Special Effects: Try other challenges" handouts for each student.
- Print out one copy of the "Math in Special Effects: Take the challenge" and the "Math in Special Effects: Try other challenges" answer keys.
- Get rulers, graph paper, chart paper, grid whiteboards, overhead transparency grids, etc. for students to record their work during the learning activities.
- Get colored stickers (optional) and colored markers, for students to mark the points and construct graphs of the Special Effects data in Learning Activities 1 & 2.

THE LESSON

INTRODUCTORY ACTIVITY

1. Begin with a brief discussion about special effects and lighting. For instance, if any of your students have used a camera or shot a video, ask them to discuss the math they have used. Ask students where they may have seen special effects visuals and what might need to be considered in terms of lighting.

2. Explain that today's lesson focuses on the use of math in Special Effects. Ask students to brainstorm how they think mathematics might be used in a filming a music video. (*Sample responses: calculating the time it takes to capture a shot, using measurement in terms of where to place the camera and the object being filmed, adjusting the camera for different settings.*)

3. Explain that today's lesson features video segments and web interactives from ***Get the Math***, a program that highlights how math is used in the real world. If this is your first time using the program with this class, you may choose to play the video segment The Setup, which introduces all the professionals and student teams featured in ***Get the Math***.

4. Introduce the video segment Math in Special Effects: Introduction by letting students know that you will now be showing them a segment from ***Get the Math***, which features Jeremy Chernick, a Special Effects designer. Ask students to watch for the math that he uses in his work and to write down their observations as they watch the video.

5. Play Math in Special Effects: Introduction**.** After showing the segment, ask students to discuss the different ways that Jeremy Chernick uses math in his work. *(Sample responses: He uses math to control the lighting while filming special effects; knowing the relationship between variables such as distance and light intensity can help Jeremy figure out how to adjust the camera settings for a better shot.)*

6. Ask students to describe the challenge that Jeremy and his colleague, , Andrew Flowers, posed to the teens in the video segment. *(The challenge is to figure out the*

relationship between distance and light intensity, and use that information to help fix a shot of an exploding flower that was underexposed.)

LEARNING ACTIVITY 1

1. Explain that the students will now have an opportunity to solve the problem, which involves using a fundamental principle of photography. The flash power, or intensity of a light source, is often measured with a light meter in a studio, but intensity readings can vary based on the distance from the light source. Understanding light intensity and how it changes, or varies, with distance can help the photographer to achieve the proper lighting for a given shot.

2. Ask students to think of situations in their daily life where they may need to apply the concept of variation between two sets of data, where one variable changes and another changes proportionally, either directly or indirectly. *(Sample responses: cost of downloading apps varies by the quantity selected; the time it takes to complete a chore or task varies by the number of people who are assisting; driving time varies with the speed of the car.)*

3. Discuss why you would need to look for the variation between light intensity and distance in this challenge. *(Sample responses: it will help to determine how to find the light intensity for any distance to adjust the shot for the music video; a model can help identify how the two sets of data are changing in relation to each other.)*

4. Review the following terminology with your students:
   - **Direct variation** means that the ratio between two variables remains constant. As one variable increases, the other variable also increases. This relationship can be represented by a function in the form $y = kx$ where $k \neq 0$.
   - **Inverse variation** means that the product of two variables remains constant. As one variable increases, the other variable decreases. This relationship can be represented by a function in the form $xy = k$ or $y = k/x$ where $k \neq 0$.
   - **Constant of variation for an inverse variation** is *k,* the product of xy for an ordered pair (x, y) that satisfies the inverse relationship.
   - **Aperture -** An opening in the lens, or circular hole, that can vary in size. It is adjusted to increase or decrease the amount of light.
   - **Diameter of aperture *(a)* -** The physical size of the hole measured through its center point. Half of the diameter is the radius of the lens.
   - **Area of aperture *(A)* -** The measurement, in square mm, of the opening of the hole formed by the aperture.
   - **Focal length *(f)* -** The distance between the optical center of the lens (typically, where the aperture is located) and the image plane, when the lens is focused at infinity.
   - **Image plane -** The fixed area behind a camera lens – inside the camera – at which the sensor or film is located, and on which pictures are focused.

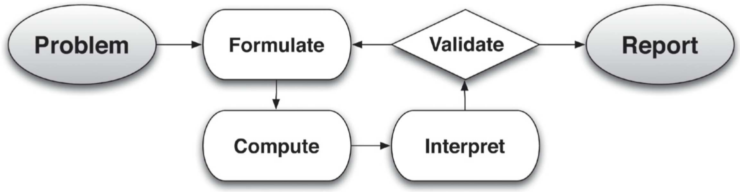
- **F-stop *(s)* -** The camera setting that regulates how much light is allowed by changing the aperture size, which is the opening of the lens.

5. Distribute the “Math in Special Effects: Take the challenge” handout. Let your students know that it is now their turn to solve the challenge that Jeremy and Andrew posed to the teams in the video. Explain that in the activity, students should use the recorded data about light intensity and distance in order to look for patterns to solve the problem.

6. Ask students to work in pairs or small groups to complete the “Math in Special Effects: Take the challenge” handout. Use the “Math in Special Effects: Take the challenge” answer key as a guide to help students as they complete the activity. (*Note: The handout is designed to be used in conjunction with the Math in Special Effects: Take the challenge activity on the website.*)
   - ***If you have access to multiple computers****, ask students to work in pairs to explore the interactive and complete the handout.*
   - ***If you only have one computer****, have students work in pairs to complete the assignment using their handouts and grid or graph paper and then ask them to report their results to the group and input their solutions into the online interactive for all to see the results.*

7. Review the directions listed on the handout.

8. As students complete the challenge, encourage them to use the following 6-step mathematical modeling cycle to solve the problem:
   - *Step 1: Understand the problem:* Identify variables in the situation that represent essential features. *(For example, light intensity, distance, and a constant of variation.)*
   - *Step 2: Formulate a model* by creating and selecting multiple representations. *(For example, students may use visual representations in sketching a graph, algebraic representations such as an equation, or an explanation/plan written in words. Student should examine their algebraic equation to identify the type of variation that is represented by the variables.)*
   - *Step 3: Compute* by analyzing and performing operations on relationships to draw conclusions. *(For example, operations include calculating the changing light intensity.)*
   - *Step 4: Interpret* the results in terms of the original situation. *(The results of the first three steps should be examined in the context of the challenge to solve real-world applications of variations, including more than just two types: direct and inverse. Inverse relationships may occur where a product of corresponding data values of any degree is constant and may involve exponential functions.)*
   - *Step 5:* Ask students to *validate* their conclusions by comparing them with the situation, and then either improving the model or, if acceptable,
   - *Step 6: Report* on the conclusions and the reasoning behind them. *(This step allows students to explain their strategy and justify their choices in a specific context.)*



Ongoing Assessment: Ask students to reflect upon the following:

- After recording the light intensity at each distance, what patterns do you notice?
- Using the data, can you identify a recursive formula that models the relationship?
- Using the data, can you identify an explicit formula that models the relationship?
- Is there only one strategy for solving the challenge? *(You may wish to have students solve graphically to determine the exponential change visually.)*

9. After students have completed the activity, ask them to share their solutions and problem-solving strategies with the class through discussion and visual materials, such as chart graph paper, grid whiteboards, overhead transparency grids, etc. Encourage students to discuss how their strategy helped (or didn't help) figure out the relationship between light intensity and distance, as well as how to adjust the camera for any shot. Ask students to discuss any difficulties they faced in completing the challenge and how they overcame those obstacles.

10. As students present their solutions, ask them to discuss the mathematics they used in solving the challenge. Ask students to describe how they selected the equation to model the relationship and any graphs used, how they calculated the values for light intensity (I), distance (d), d^2, and k, why rounding was important, and the strategy they would recommend to Jeremy and Andrew for finding the light intensity for any distance in a high-speed special effects shot.

11. Introduce the Math in Special Effects: See how the teams solved the challenge video segment by letting students know that they will now be seeing how the teams in the video solved the Special Effects challenge. Ask students to observe what strategies the teams used and whether they are similar to or different from the strategies presented by the class.

12. Play Math in Special Effects: See how the teams solved the challenge. After showing the video, ask students to discuss the strategies the teams used and to compare them to the strategies used by the class. How are they similar? How are they different? During the discussion, point out that the two teams in the video solved the Special Effects challenge in two distinct ways. Discuss the strategies listed in the "Math in Special Effects: Take the challenge" answer key, as desired.

LEARNING ACTIVITY 2:

1. Go to the Math in Special Effects: Try other challenges interactive. Explain to your students that they will use the web interactive to solve a series of problems similar to the one Jeremy Chernick presented in the video segment. In this multi-level activity,

students are challenged to learn how a camera setting called an f-stop affects the amount of light coming through the lens and use the information to improve a shot. *Note: As in Learning Activity 1, you can conduct this activity with one computer and an LCD projector in front of the entire class or your students can work in small groups on multiple computers. This can also be assigned to students to complete as an independent project or homework using the accompanying handout as a guide.*

2. Distribute the "Math in Special Effects: Try other challenges" handout. Clarify and discuss the directions.

3. Ask students to complete the handout as they explore the online challenges. *Note: If you are using one computer, have your students work in pairs, taking turns inputting their responses into the web interactive to test their choices as they determine how focal length, aperture, and f-stop are related; how much light passes through at a given f-stop, the relationship between f-stop and area of the aperture, and how to use an understanding of f-stops to adjust the camera setting for a given shot.*

4. As in Learning Activity 1, encourage your students to use the 6-step mathematical modeling cycle as they develop a strategy to solve the challenges.

5. After students have completed the activity, lead a group discussion and encourage students to share their strategies and solutions to the challenges. *(Sample responses: There is a relationship between f-stops and light intensity (or area of the aperture). "Opening up" to the next lower f-stop (for instance, going from f/1.4 to f/1) lets in twice as much light by increasing the diameter of the opening by a factor of* $\sqrt{2}$*, or about 1.414. As diameter increases by a factor of* $\sqrt{2}$*, the area of the aperture, and therefore, light intensity, doubles. Conversely, "closing down" to each greater f-stop (for instance, going from f/2 to f/2.8) reduces the light intensity by half.)*

CULMINATING ACTIVITY

1. Assess deeper understanding: Ask your students to reflect upon and write down their thoughts about the following:

   - How did you determine an effective strategy for solving the challenges in this lesson? What are your conclusions and the reasoning behind them?

   - Compare and contrast the various algebraic and graphical representations possible for the problem. How does the approach used to solve the challenge affect the choice of representations? (*Sample responses: attempting to estimate the direction and speed of the variation, or change, between two sets of data with a quick visual can be determined by using a graph or pictorial model; an approach that attempts to show a proportional relationship by quantifying the variability, or change, would be best represented with an algebraic model.*)

- Why is it useful to represent real-life situations algebraically? *(Sample responses: Using symbols, graphs, and equations can help visualize solutions when there are situations that require inverse relationships.)*

- What are some ways to represent, describe, and analyze patterns that occur in our world? *(Sample responses: patterns can be represented with graphs, expressions, and equations to show and understand changes between two sets of data such as light intensity and distance.)*

2. After students have written their reflections, lead a group discussion where students can discuss their responses. During the discussion, ask students to share their thoughts about how algebra can be applied to the world of Special Effects. Ask students to brainstorm other real-world situations which involve the type of math and problem solving that they used in this lesson. *(Sample responses: there is an inverse relation between the temperature of the water in a lake or ocean and the depth where the measurement is taken; the speed of a bicycle gear is inversely proportional to the number of teeth in the gear.)*

LEARNING STANDARDS

Common Core State Standards 2010
[Note: You may also wish to view Pathways 1 and 2 for Algebra I connections in the CCSS]

Mathematical Practices
- Make sense of problems and persevere in solving them
- Reason abstractly and quantitatively
- Construct viable arguments and critique the reasoning of others
- Model with mathematics
- Use appropriate tools strategically
- Attend to precision
- Look for and make use of structure
- Look for and express regularity in repeated reasoning

Algebra Overview
- Seeing Structure in Expressions
  A.SSE.1a, 1b, 2 Interpret the structure of expressions.
  A.SSE.3a, 3b Write expressions in equivalent forms to solve problems.
- Arithmetic with Polynomials and Rational Functions
  A.APR.1 Perform arithmetic operations on polynomials.
- Creating Equations
  A.CED.2, 4 Create equations that describe numbers or relationships.
- Reasoning with Equations and Inequalities
  A.REI.1 Understand solving equations as a process of reasoning and explain the reasoning.

- Represent and solve equations and inequalities graphically
  A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Functions Overview

- Interpreting Functions
  F.IF. 1, 2 Understand the concept of a function and use function notation.
  F.IF.4, 5, 6 Interpret functions that arise in applications in terms of a context.
- Analyzing Functions using different representations
  F.IF.7e. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases (i.e., exponential).
- Building Functions
  F.BF.1 Build a function that models a relationship between two quantities.
- Build new functions from existing functions
  F-BF 4a, 4b, 4c. Find inverse functions:
  a. Solve and equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse.
  b. Verify by composition that one function is the inverse of another.
  c. Read values of an inverse function from a graph or a table, given that the function has an inverse.

Geometry Overview

- Modeling with Geometry
  Apply geometric concepts in modeling situations:
  G.MG.1. Use geometric shapes, their measures, and their properties to describe objects.

Modeling Standards

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice.

Name:________________________________ Date:__________________

Math in Special Effects: Take the Challenge

Student Handout

Jeremy Chernick, a designer at J & M Special Effects, uses math in his work when creating high-speed effects for music videos. Recently, Freelance Whales required special effects while filming the video for their single, "Enzymes."

Your challenge is to:

- Figure out the relationship between distance and light intensity;
- Model your data using a mathematical representation to determine how to find the light intensity for any distance; then,
- Use this information to help fix a shot of an exploding flower that was underexposed.

This activity is designed to be used in conjunction with the online interactive. Go to www.getthemath.org, click on "The Challenges," then scroll down and click on "Math in Special Effects: Take the Challenge."

A. RECORD THE LIGHT INTENSITY AT VARIOUS DISTANCES

Identify what you already know. Use the chart on the last page of this handout to record information found in the interactive.

- The two sets of data displayed in the chart are:
  ______________________ and ______________________
- As you move the silhouette across the screen, record the light intensity (measured in lumens ranging from 0 to 1) at each distance from 30″ to 180″ in the first two columns of the chart.

B. REPRESENT THE RELATIONSHIP BETWEEN LIGHT INTENSITY AND DISTANCE

1. **Plan it out.** Describe the strategy you plan to use to find and represent the relationship between light intensity and distance.

2. **Model your data** by identifying an equation you think best represents the relationship. Explain your reasoning as to why you chose this equation.

3. Solve for the variables using the equation you selected and your recorded data. Show all work below. Be sure to record the missing values in your chart to test out your prediction. Remember to round your values based on the rule specified in the interactive.

4. Validate your answer: Is your equation a good representation of the relationship?

- If not, try looking for a pattern to determine how to find the light intensity for any distance, and try a different equation.
- If so, explain the kind of mathematical relationship that exists between distance and light intensity.

C. HOW MUCH DO YOU NEED TO ADJUST THE DISTANCE TO FIX THE SHOT?

1. **Interpret your solution in the context of the special effects problem.**
To fix the original shot, *the light intensity needs to be 4 times greater* than it was in the original shot. Based on the relationship you found, how should you adjust the distance from the light source?

2. Explain your reasoning. If you were going to email Jeremy to explain your strategy for finding the correct light intensity at any distance, what would you tell them?

REPRESENT THE RELATIONSHIP BETWEEN LIGHT INTENSITY AND DISTANCE

Distance (d)	Light Intensity (I)	__________ (__)	Constant (k) (round to nearest________)
30”	1		
60”			
90”			
120”			
150”			
180”			

- The light intensity = 1 at the original distance (measured in lumens from 0 to 1).
- Record the light intensity (I) at each distance (d) from the light source.
- Model your data by identifying an equation you think best represents the relationship. Write the equation here:

  ____

- Solve for the missing values and record in the chart to test out your prediction.

Name:__________________________________ Date:___________________

Math in Special Effects: Try Other Challenges

Student Handout

When filming special effects, a high-speed photographer needs to control the duration and impact of light by adjusting a number of settings, including frame rate, shutter angle, and f-stop. Knowing the relationship between these variables can help you figure out how to get the best shot.

Your challenge is to learn how a camera setting, called an f-stop, affects the amount of light coming through the lens and use your understanding of it to improve your shot.

This activity is designed to be used in conjunction with the online interactive. Go to www.getthemath.org, click on "The Challenges," then scroll down and click on Math in Special Effects: Try other challenges."

A. SELECT A LENS SIZE

☐ **35 mm** ☐ **50 mm** ☐ **200 mm**

B. RECORD F-STOP SETTINGS

1. **Identify what you already know.** Look at the chart that corresponds to the lens you selected (on the last pages of this handout). View "Photo Fundamentals" for definitions.
   - The two sets of data displayed in the chart are:
     ________________________ and ________________________
   - Following the instructions in the interactive, record the f-stop value that corresponds to the focal length *(f)* and diameter of the aperture *(a)* for each row in the chart. *(Note: f-stops are rounded to the nearest tenth.)*

C. HOW ARE FOCAL LENGTH, APERTURE, AND F-STOP RELATED?

1. **Plan it out.** Describe your strategy for identifying the relationship represented by the f-stop *(s)* in terms of focal length *(f)* and diameter of the aperture *(a).*

2. **Model your data** by identifying an equation you think best represents the relationship. Explain your reasoning as to why you chose this equation.

3. Solve for the variables using the equation you selected and your recorded data. Show all work below. Be sure to record the corresponding values in your chart to test out your prediction.

4. Validate your answer: Is your equation a good representation of the relationship?

- If not, try looking for a pattern to determine how to find the matching f-stop(s) and try another equation.
- If so, explain how the aperture diameter and f-stop are related.

D. HOW MUCH LIGHT PASSES THROUGH AT A GIVEN F-STOP?

1. Plan it out. Identify the formula you will use for calculating the area, ***A,*** of the aperture opening for each f-stop.

2. Solve by calculating ***A*** for each row in your chart. (Use π = 3.14 and round to the nearest sq mm for 200mm lens or the nearest tenth for 50mm or 35mm lens.) Use the space below to show your work. **Record your solutions** in each row of the chart.

E. IDENTIFY THE RELATIONSHIP BETWEEN F-STOP *(s)* AND AREA OF APERTURE *(A)*

1. **Plan it out.** Describe your strategy for identifying the relationship between f-stop *(s)* and Area of aperture *(A).*

2. **Model your data** by identifying an equation you think best represents the relationship. Why did you choose this equation?

3. **Solve** for the constant, ***k,*** for each row. Show all work below. Be sure to record the values for **(*k*)** in your chart to test out your prediction. *(Round to the nearest thousand.)*

4. **Validate your answer:** Is your equation a good representation of the relationship?
    - If not, try looking for a pattern to determine how to find the constant value **(*k*)** and try another equation.
    - If so, explain how the f-stop and Area of the aperture are related.

F. USE YOUR UNDERSTANDING OF F-STOPS TO ADJUST THIS SHOT

1. The f-stop is currently set at f/5.6, but the shot is underexposed. If the shot requires a setting that is 8 times brighter, to what f-stop should you adjust to get more light?

2. If the shot requires a setting that is 1/4 as bright and the f-stop is currently set at f/5.6, to what f-stop should you adjust to get less light?

3. If the shot requires a setting that is 8 times brighter and the f-stop is currently set at f/16, to what f-stop should you adjust to get more light?

4. **Explain your reasoning.** If you were going to email a friend to explain your strategy for finding the correct f-stop for any shot, what would you tell them?

Photo Fundamentals	
Aperture	An opening in the lens, or circular hole, that can vary in size. It is adjusted to increase or decrease the amount of light.
Diameter of aperture *(a)*	The physical size of the hole measured through its center point. Half of the diameter is the radius of the lens.
Area of aperture *(A)*	The measurement, in square mm, of the opening of the hole formed by the aperture.
Focal length *(f)*	The distance between the optical center of the lens (typically, where the aperture is located) and the image plane, when the lens is focused at infinity.
Image plane	The fixed area behind a camera lens – inside the camera – at which the sensor or film is located, and on which pictures are focused.
f-stop *(s)*	The camera setting that regulates how much light is allowed by changing the aperture size, which is the opening of the lens.

focal length (*f*) **35 mm lens**	diameter of aperture (*a*)	f-stop (*s*) (round to nearest tenth)	Area of aperture (*A*) (in sq. mm to nearest tenth)	Constant (*k*) (round to nearest thousand)
35	35			
35	25			
35	17.5			
35	12.5			
35	8.8			
35	6.3			
35	4.4			

focal length (*f*) **50 mm lens**	diameter of aperture (*a*)	f-stop (*s*) (round to nearest tenth)	Area of aperture (*A*) (in sq. mm to nearest tenth)	Constant (*k*) (round to nearest thousand)
50	50			
50	35.7			
50	25			
50	17.9			
50	12.5			
50	8.9			
50	6.25			

focal length *(f)* **200 mm lens**	diameter of aperture (*a*)	f-stop *(s)* (round to nearest tenth)	Area of aperture *(A)* (in sq. mm to nearest integer)	Constant *(k)* (round to nearest thousand)
200	200			
200	142.9			
200	100			
200	71.4			
200	50			
200	35.7			
200	25			

Name:________________________________ Date:_________________

Math in Special Effects: Take the Challenge

Answer Key

Jeremy Chernick, a designer at J & M Special Effects, uses math in his work when creating high-speed effects for music videos. Recently, Freelance Whales required special effects while filming the video for their single, "Enzymes."

Your challenge is to:

- Figure out the relationship between distance and light intensity;
- Model your data using a mathematical representation to determine how to find the light intensity for any distance; then,
- Use this information to help fix a shot of an exploding flower that was underexposed.

This activity is designed to be used in conjunction with the online interactive. Go to www.getthemath.org, click on "The Challenges," then scroll down and click on "Math in Special Effects: Take the Challenge."

A. RECORD THE LIGHT INTENSITY AT VARIOUS DISTANCES

Identify what you already know. Use the chart on the last page of this handout to record information.

- The two sets of data displayed in the chart are:
  Distance (d) and **Light Intensity (I)**
- As you move the silhouette across the screen, record the light intensity (measured in lumens ranging from 0 to 1) at each distance from 30" to 180" in the first two columns of the chart. **(See chart on last page.)**

B. REPRESENT THE RELATIONSHIP BETWEEN LIGHT INTENSITY AND DISTANCE

1. **Plan it out.** Describe the strategy you plan to use to find and represent the relationship between light intensity and distance.

   Possible plan: **Look at each increasing distance and see if the light intensity is changing in the same way. If not, how is it changing?**

   Possible strategies: **After recording the distance and the light intensity, teams may identify the relationship as a product first.**

   A: Using the data, identify a recursive formula such that each time, the intensity is the product (k) divided by d^2.

   B: Using the data, identify an explicit formula using the fact that the intensity is decreasing by $1/d^2$ each time the distance is increased (2x, 3x, 4x, 5x, 6x).

   C. Using a graph (or scatter plot), students may identify above relationship.

2. **Model your data** by identifying an equation you think best represents the relationship. Explain your reasoning as to why you chose this equation.

   $$I = \frac{k}{d^2}$$

3. **Solve** for the variables using the equation you selected and your recorded data. Show all work below. Be sure to record the missing values in your chart to test out your prediction. Remember to round your values based on the rule specified in the interactive.
   See completed chart on last page.

4. **Validate your answer:** Is your equation a good representation of the relationship?
   - If not, try looking for a pattern to determine how to find the light intensity for any distance, and try a different equation.
   - If so, explain the kind of mathematical relationship that exists between distance and light intensity.

   As the distance increases, the intensity of the light source drops off quickly, or decreases, as an inverse relationship, specifically, an inverse square relationship.

C. HOW MUCH DO YOU NEED TO ADJUST THE DISTANCE TO FIX THE SHOT?

1. **Interpret your solution in the context of the special effects problem.**
   To fix the original shot, *the light intensity needs to be 4 times greater* than it was in the original shot. Based on the relationship you found, how should you adjust the distance from the light source?

   If $I = \frac{k}{d^2}$, then $4I = \frac{k}{?d^2}$?

   Multiplying both sides of the original equation by 4, you have:

   $$4I = 4 * \frac{k}{d^2} = \frac{k}{\left(\frac{1}{4}\right)d^2} = \frac{k}{\left(\frac{1}{2}d\right)^2}$$

   So, you would reduce the distance by ½.

2. **Explain your reasoning.** If you were going to email Jeremy to explain your strategy for finding the correct light intensity at any distance, what would you tell them?
   Possible response: You can find the correct light intensity by adjusting (increasing or decreasing) it by the product of a constant and $1/d^2$.

REPRESENT THE RELATIONSHIP BETWEEN LIGHT INTENSITY AND DISTANCE

Distance (d)	Light Intensity (I)	d^2	Constant (k) (round to nearest hundred)	Students may notice the following relationships:
30"	1	900	900 ≈ 900	Original I
60"	0.241	3,600	867.6 ≈ 900	2d ≈ $\frac{1}{4}$ original intensity
90"	0.117	8,100	947.7 ≈ 900	3d ≈ 1/9 original intensity
120"	0.060	14,400	864 ≈ 900	4d ≈ 1/16 original intensity
150"	0.040	22,500	900 ≈ 900	5d ≈ 1/25 original intensity
180"	0.029	32,400	939.6 ≈ 900	6d ≈ 1/36 original intensity

- The light intensity = 1 at the original distance (measured in lumens from 0 to 1).
- Record the light intensity (I) at each distance (d) from the light source.
- Model your data by identifying an equation you think best represents the relationship. Write the equation here: $I = \frac{k}{d^2}$
- Solve for the missing values and record in the chart to test out your prediction.

Name:___________________________________ Date:___________________

Math in Special Effects: Try Other Challenges

Answer Key

When filming special effects, a high-speed photographer needs to control the duration and impact of light by adjusting a number of settings, including frame rate, shutter angle, and f-stop. Knowing the relationship between these variables can help you figure out how to get the best shot.

Your challenge is to learn how a camera setting, called an f-stop, affects the amount of light coming through the lens and use your understanding of it to improve your shot.

This activity is designed to be used in conjunction with the online interactive. Go to www.getthemath.org, click on "The Challenges," then scroll down and click on "Math in Special Effects: Try other challenges."

A. SELECT A LENS SIZE

☐ **35 mm** ☐ **50 mm** ☐ **200 mm**

B. RECORD F-STOP SETTINGS

1. **Identify what you already know.** Look at the chart that corresponds to the lens you selected (on the last page of this handout). View "Photo Fundamentals" for definitions.
   - The two sets of data displayed in the chart are:
     Focal length *(f)* and diameter of aperture *(a)*
   - Following the instructions in the interactive, record the f-stop value that corresponds to the fixed focal length *(f)* and diameter of the aperture *(a)* for each row in the chart. *(Note: f-stops are rounded to the nearest tenth.)*

C. HOW ARE FOCAL LENGTH, APERTURE, AND F-STOP RELATED?

1. **Plan it out.** Describe your strategy for identifying the relationship represented by the f-stop *(s)* in terms of focal length *(f)* and diameter of the aperture *(a)*.
   Students should examine the data and look for a pattern. They may notice that the f-stop is a ratio of the focal length to the diameter of the aperture.

2. **Model your data** by identifying an equation you think best represents the relationship. Explain your reasoning as to why you chose this equation.

$$s = \frac{f}{a}$$

3. **Solve** for the variables using the equation you selected and your recorded data. Show all work below. Be sure to record the corresponding values in your chart to test out your prediction.
   (See charts for all solutions.)

4. **Validate your answer:** Is your equation a good representation of the relationship?
   - If not, try looking for a pattern to determine how to find the matching f-stop(s) and try another equation.
   - If so, explain how the aperture diameter and f-stop are related.

   There is an inverse relationship between f-stop and the diameter of aperture.

D. HOW MUCH LIGHT PASSES THROUGH AT A GIVEN F-STOP?

1. Plan it out. Identify the formula you will use for calculating the area, ***A,*** of the aperture opening for each f-stop.

2. Solve by calculating ***A*** for each row in your chart. *(Use* π = 3.14 and round to the nearest sq mm for 200mm lens or the nearest tenth for 50mm or 35mm lens.) Use the space below to show your work. **Record your solutions** in each row of the chart.

(See charts for all solutions.)

E. IDENTIFY THE RELATIONSHIP BETWEEN F-STOP *(s)* AND AREA OF APERTURE *(A)*

1. **Plan it out.** Describe your strategy for identifying the relationship between f-stop *(s)* and Area of aperture *(A).*
   Students should examine the data to look for a pattern. They may notice that there is a relationship between f-stops and light intensity (or area of the aperture). "Opening up" to the next lower f-stop (for instance, going from f/1.4 to f/1) lets in twice as much light by increasing the diameter of the opening by a factor of $\sqrt{2}$, or about 1.414. As diameter

   increases by a factor of $\sqrt{2}$, the area of the aperture, and therefore, light intensity,

   doubles. Conversely, "closing down" to each greater f-stop (for instance, going from f/2 to f/2.8) reduces the light intensity by half.

2. **Model your data** by identifying an equation you think best represents the relationship. Why did you choose this equation?

   $A = \frac{k}{s^2}$

3. **Solve** for the constant, ***k,*** for each row. Show all work below. Be sure to record the values for **(*k*)** in your chart to test out your prediction. *(Round to the nearest thousand.)*
   (See charts for all solutions.)

4. Validate your answer: Is your equation a good representation of the relationship?

- If not, try looking for a pattern to determine how to find the constant value ***(k)*** and try another equation.
- If so, explain how the f-stop and Area of the aperture are related.

There is an inverse square relationship between f-stop (s) and the Area of aperture (A).

F. USE YOUR UNDERSTANDING OF F-STOPS TO ADJUST THIS SHOT

1. The f-stop is currently set at f/5.6, but the shot is underexposed. If the shot requires a setting that is 8 times brighter, to what f-stop should you adjust to get more light?
   Open up 3 stops to f/2
2. If the shot requires a setting that is 1/4 as bright and the f-stop is currently set at f/5.6, to what f-stop should you adjust to get less light?
   Close down 2 stops to f/11
3. If the shot requires a setting that is 8 times brighter and the f-stop is currently set at f/16, to what f-stop should you adjust to get more light?
   Open up 3 stops to f/5.6

4. **Explain your reasoning.** If you were going to email a friend to explain your strategy for finding the correct f-stop for any shot, what would you tell them?

There is a relationship between f-stops and light intensity (or area of the aperture). Opening up to the next lower f-stop lets in twice as much light by increasing the diameter of the opening by a factor of $\sqrt{2}$, or about 1.414. As diameter increases by a factor of $\sqrt{2}$, the area of the aperture, and therefore, light intensity, doubles. Conversely, closing down to each greater f-stop reduces the light intensity by half.

Photo Fundamentals	
Aperture	An opening in the lens, or circular hole, that can vary in size. It is adjusted to increase or decrease the amount of light.
Diameter of aperture *(a)*	The physical size of the hole measured through its center point. Half of the diameter is the radius of the lens.
Area of aperture *(A)*	The measurement, in square mm, of the opening of the hole formed by the aperture.
Focal length *(f)*	The distance between the optical center of the lens (typically, where the aperture is located) and the image plane, when the lens is focused at infinity.
Image plane	The fixed area behind a camera lens – inside the camera – at which the sensor or film is located, and on which pictures are focused.
f-stop *(s)*	The camera setting that regulates how much light is allowed by changing the aperture size, which is the opening of the lens.

Solutions for 35 mm lens

focal length *(f)* **35 mm lens**	diameter of aperture *(a)*	f-stop *(s)* (round to nearest tenth)	Area of aperture *(A)* (in sq. mm to nearest tenth)	Constant *(k)* (round to nearest thousand)
35	35	**1.0**	**961.6**	**1,000**
35	25	**1.4**	**490.6**	**1,000**
35	17.5	**2.0**	**240.4**	**1,000**
35	12.5	**2.8**	**122.7**	**1,000**
35	8.8	**4.0**	**60.8**	**1,000**
35	6.3	**5.6**	**31.2**	**1,000**
35	4.4	**8.0**	**15.2**	**1,000**

Solutions for 50 mm lens

focal length (*f*) **50 mm lens**	diameter of aperture (*a*)	f-stop (*s*) (round to nearest tenth)	Area of aperture (*A*) (in sq. mm to nearest tenth)	Constant (*k*) (round to nearest thousand)
50	50	**1.0**	**1,962.5**	**2,000**
50	35.7	**1.4**	**1,000.5**	**2,000**
50	25	**2.0**	**490.6**	**2,000**
50	17.9	**2.8**	**251.5**	**2,000**
50	12.5	**4.0**	**122.7**	**2,000**
50	8.9	**5.6**	**62.2**	**2,000**
50	6.25	**8.0**	**30.7**	**2,000**

Solutions for 200 mm lens

focal length (*f*) **200 mm lens**	diameter of aperture (*a*)	f-stop (*s*) (round to nearest tenth)	Area of aperture (*A*) (in sq. mm to nearest integer)	Constant (*k*) (round to nearest thousand)
200	200	**1.0**	**31,400**	**31,000**
200	142.9	**1.4**	**16,030**	**31,000**
200	100	**2.0**	**7,850**	**31,000**
200	71.4	**2.8**	**4,002**	**31,000**
200	50	**4.0**	**1,963**	**31,000**
200	35.7	**5.6**	**1,000**	**31,000**
200	25	**8.0**	**491**	**31,000**